MTTC Secondary Math Review

1) A student is asked to prove "$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all positive integers n" using mathematical induction. The student first proves the statement $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for $n = 1$. What is the next step?

   a) Assume statement is false for $n = k+1$ and prove it is true for $n = k$.

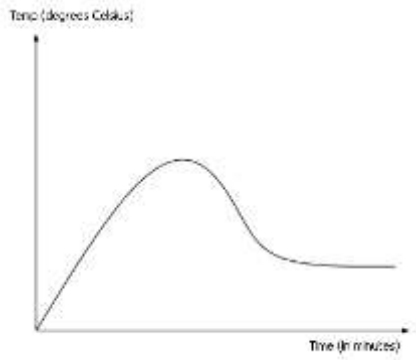
   b) Assume statement is false for $n = k$ and prove it is true for $n = k+1$.

   c) Assume statement is true for $n = k+1$ and prove it is true for $n = k$.

   d) Assume statement is true for $n = k$ and prove it is true for $n = k+1$.

2) You place a frozen pizza in the oven and let it bake for 30 minutes. After you take it out of the oven you let it cool before you eat it. What would a rough graph of the temperature of the pizza as a function of time look like?

a)



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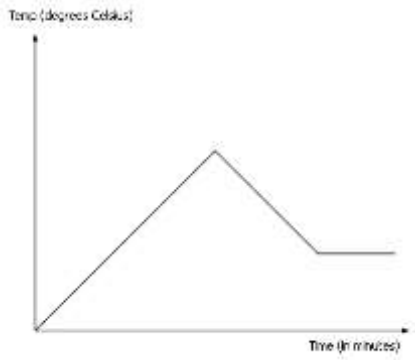

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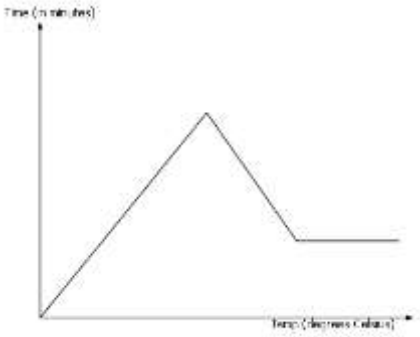

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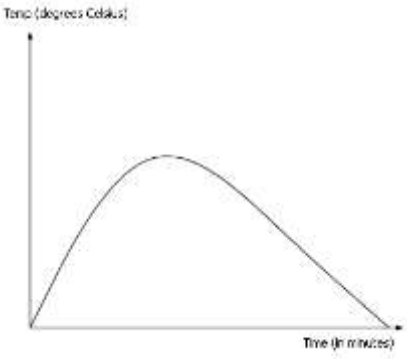


3) If $f(x) = 3x^2 - 4x$ and $g(x) = 2x + 3$, find $(f \circ g)(-2)$.

   a) -1 b) 7 c) 20 d) 41

4) Given $f(x) = x \sin x$, which equation is equivalent to $f'(x) = 0$?

   a) $x = -\cot x$

   b) $x = -\tan x$

   c) $\cos x = 0$

   d) $x \cdot \tan x = 0$

e)

5) Which of the following matrices solves the matrix equation $Ax = b$, where $x = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $b = \begin{pmatrix}10\\16\\6\end{pmatrix}$?

a) $\begin{bmatrix}1 & 2 & 4\\3 & 2 & 1\\1 & 1 & 1\end{bmatrix}$

b) $\begin{bmatrix}2 & 1 & 2\\4 & 2 & 2\\1 & 1 & 3\end{bmatrix}$

c) $\begin{bmatrix}1 & 3 & 1\\2 & 1 & 4\\1 & 1 & 1\end{bmatrix}$

d) $\begin{bmatrix}3 & 2 & 1\\1 & 2 & 3\\1 & 1 & 2\end{bmatrix}$

6) The edge of a cube was measured to be 5 cm. The possible error in measurement is 0.1 cm. Using differentials determine the relative percent error in computing the volume of the cube.

a) 750%

b) 60%

c) 125%

d) 6%

7) Find the limit of the following function. $\lim_{x\to\infty} \frac{3x^2+5x}{x^3+4x^2}$

a) Limit does not exist

b) ∞

c) 1

d) 0

8) Given $f(x) = 3x^2 + 4x + 2$ what is the equation of the tangent line at $x = 3$?

a) $y = 3x - 2$

b) $y = 22x - 25$

c) $y = 20x + 14$

d) $y = x + 41$

9) Find the solutions to the equation $x^2 + 2x + 2 = 0$.

a) $x = -1,1$

b) $x = -1 - i, -1 + i$

c) $x = -2 - 2i, -1 - i$

d) $x = 1 - i, 1 + i$

10) Solve the following system of equations.

$$\begin{cases} 3x + 2y + z = 2 \\ 2x - y - z = 1 \\ x + y = 1 \end{cases}$$

a) $x = 2, y = 1, z = 1$

b) $x = 1, y = 1, z = -3$

c) $x = \frac{1}{2}, y = 1, z = -\frac{1}{2}$

d) $x = \frac{1}{2}, y = \frac{1}{2}, z = -\frac{1}{2}$

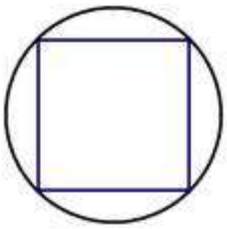
11) What is the area of a square inscribed into a circle whose diameter is 1 m?

a) $\frac{1}{3}$ meters^2

b) $\frac{1}{3}\pi$ meters^2

c) $\frac{1}{2}\pi$ meters^2

d) $\frac{1}{2}$ meters^2



12) Find $f(1),\ f(2),\ f(3)$ and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 3$ and $f(n+1) = 3f(n) - 2$ for $n = 1,2,3$ ....

a) $f(1) = -2, f(2) = -8, f(3) = -26, f(4) = -80$

b) $f(1) = 7, f(2) = 19, f(3) = 55, f(4) = 163$

c) $f(1) = 1, f(2) = 4, f(3) = 7, f(4) = 10$

d) None of the above

13) Find the inverse of $f(x) = \frac{4x-1}{3-2x}$.

a) $f^{-1}(x) = \frac{3y+1}{2y+4}$

b) $f^{-1}(x) = \frac{2x+4}{3x+1}$

c) $f^{-1}(x) = \frac{3x+1}{2x+4}$

d) $f^{-1}(x) = \frac{2y+4}{3y+1}$

14) A frog attempts to leap across a small pond. On his first jump, he jumps $\frac{5}{8}$ of the total distance. Every jump thereafter, he only manages to jump $\frac{5}{8}$ of the remaining distance. Which of the following sums describes the distance that the frog has jumped?

a) $1 - \left[\left(\frac{5}{8}\right) + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \cdots\right]$

b) $\frac{5}{8}\left[1 + \frac{3}{8} + \left(\frac{3}{8}\right)^2 + \left(\frac{3}{8}\right)^3 + \cdots\right]$

c) $\frac{5}{8} + \frac{5}{16} + \frac{5}{32} + \frac{5}{64} + \cdots$

d) $\frac{5}{8} + \left(\frac{5}{8}\right)^2 + \left(\frac{5}{8}\right)^3 + \left(\frac{5}{8}\right)^4 + \cdots$

15) What is the ratio of surface area to volume of a sphere with radius of 3?

a) 1:1

b) 1:3

c) 3:1

d) 2:1

16) What is the value of $f(x) = \cos^{-1}\left(2 - \sqrt{2}\sin(x)\right)$ when $x = \frac{\pi}{4}$?

a) $\frac{1}{2}$

b) $\frac{1}{4}$

c) 0

d) $\frac{\pi}{4}$

17) Below is a sequence of numbers;

2, 3, 5, 8, 13, 21, …

If this pattern continues, how many prime numbers will there be in the first 10 terms of the sequence?

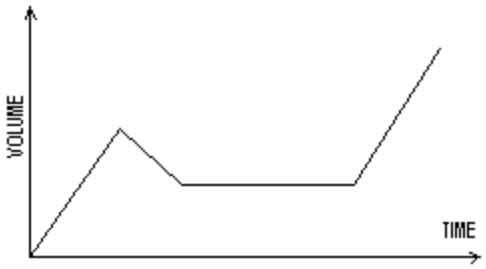
a) 3

b) 2

c) 5

d) 4

18) A small swimming pool is being filled using a garden hose. The following graph depicts the volume of water in the pool with respect to time. Which of the following statement best describe the effort to fill the pool?



a) Filling increased steadily for a time, decreased, went at a steady rate, and then filled more quickly than at the start.

b) Filling increased at a steady rate for a time, stopped, increased at a steady rate, and then stopped again.

c) Filling went at a steady rate for a time, the pool began to lose water, filling and emptying stopped for a time, and then filled again at a steady rate.

d) Filling began steadily. Slowed down, went at a steady rate, and then stopped completely.

19) The density of a right circular cone with a height of 8 cm and diameter of 5 cm is 2.6 g/cm^3. What is its mass?

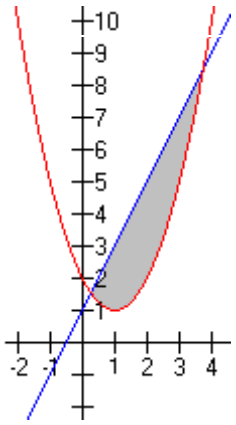
a) $\frac{160}{3}\pi$ grams

b) 60 π grams

c) $\frac{130}{3}\pi$ grams

d) $\frac{100}{3}\pi$ grams

20) Given the graphs: $f(x) = 2x + 1$ and $g(x) = x^2 - 2x + 2$, which of the polynomial functions models the vertical distance between the two functions, specified by the shaded region in the figure?



a) $h(x) = -x^2 + 4x - 1$

b) $h(x) = -x^2 + 3$

c) $h(x) = x^2 - 4x + 1$

d) $h(x) = -x^2 - 1$

21) In logic, the statement $P \wedge (\sim P)$ is an example of

a) A tautology

b) An implication

c) A contradiction

d) An equivalence

22) Suppose we wish to find the lowest degree non-constant polynomial that passes through the points (–2,1), (0,1), (3,1), and (5,1). Which of the following polynomials satisfies the condition?

a) $(x-1)(x+1)(x-4)(x-6)$

b) $x(x+2)(x-3)(x-5)$

c) $x(x+2)(x-3)(x-5)-1$

d) $x(x+2)(x-3)(x-5)+1$

23) Suppose the following statement is true: If it is raining outside, then Jethro carries his umbrella. Which of the following statements is definitely true?

a) If it is not raining outside, Jethro does not carry his umbrella.

b) If Jethro carries his umbrella, it is raining outside.

c) If Jethro does not carry his umbrella, then it is not raining outside.

d) If it is not raining outside, Jethro carries his umbrella.

24) Which of the following is <u>not</u> a possible rational root of the polynomial $100x^6 + 21x^5 - 40x^3 - 3x^2 + 17x - 100$?

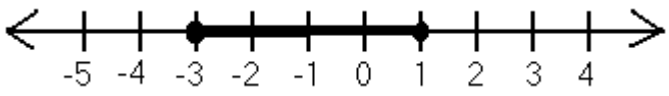
a) $\frac{5}{2}$

b) $\frac{3}{4}$

c) 20

d) $\frac{4}{5}$

25) Consider the solution set shown on the number line below.



Which of the following absolute value inequalities has the same solution set as the one shown?

a) $|x-1| \leq 2$

b) $|x+1| \geq 2$

c) $|x+1| \leq 2$

d) $|x-1| \geq 2$

26) Suppose that when a dry sponge is immersed in water, each side increases by a linear factor of 3. Also, when the immersed sponge is taken out of the water, each side shrinks by a linear factor of 2. How does the volume of a wet sponge (after pulled out of the water) compare to the volume of a dry sponge?

a) Increased by a factor of $\frac{3}{2}$

b) Increased by a factor of $\frac{9}{4}$

c) Increased by a factor of $\frac{27}{8}$

d) Increased by a factor of $\frac{9}{2}$

27) Suppose a satellite orbiting 200 miles above the earth's surface is traveling 1090 mi/hr. Given that the earth's radius is 3959 miles, how long will it take the satellite to make one complete orbit? Assume that the earth is a sphere. (round to the nearest hour).

a) 23 hrs

b) 13 hrs

c) 4 hrs

d) 24 hrs

28) The graph of the function, $f(x) = 3x^9 - 4x^2 + 6$ has:

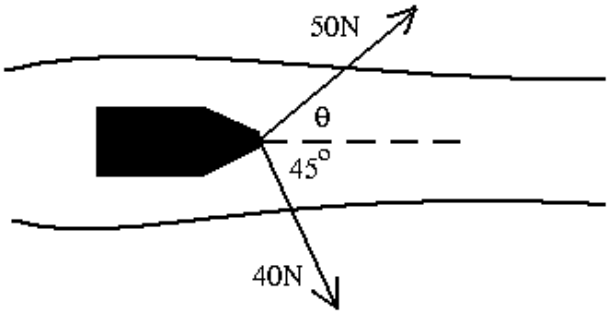
a) Origin symmetry

b) x- axis symmetry

c) y-axis symmetry

d) no symmetry

29) A boat is being pulled down a river by two ropes, as shown in the figure. One rope is pulled at 45° from the river with a force of 40 N. The other rope is pulled with a force of 50 N. Find the angle at which the second rope must be pulled so that the boat goes straight down the river, rounded to the nearest degree.



a) 45°

b) 51°

c) 34°

d) 30°

30) A walker and biker leave from the same place at the same time. The walker travels due east at a steady rate of 2mph and the biker travels northwest (N45°W) at a steady rate of 7 mph. Which equation must be solved to determine the distance between the two after 2 hours?

a) $4^2 + 14^2 = c^2$

b) $d = \tan^{-1}\left(\frac{7}{2}\right)$

c) $a = \sqrt{4^2 + 14^2 - 2(4)(14)\cos 135°}$

d) $d = \cos^{-1}\left(\frac{2}{7}\right)$

31) A piece of paper with length l and thickness t is folded repeatedly folded in half (lengthwise). As the folding continues, the length _____ exponentially and the thickness _____ exponentially.

a) Increases; Increases

b) Increases; Decreases

c) Decreases; Increases

d) Decreases; Decreases

32) Suppose a solid sphere is dissolved in a liquid. The sphere is made of a soluble material such that the surface dissolves at a rate of $2\ {}^{\text{cm}^2}/_{\text{sec}}$. How fast is the volume of the sphere changing when the radius is 4 cm?

a) $2\ {}^{\text{cm}^3}/_{\text{sec}}$

b) $4\ {}^{\text{cm}^3}/_{\text{sec}}$

c) $6\ {}^{\text{cm}^3}/_{\text{sec}}$

d) $8\ {}^{\text{cm}^3}/_{\text{sec}}$

33) A ball is thrown off the top of a 100 ft building. Its vertical distance, measured in feet, from the ground is given by $y(t) = 10t - 5t^2 + 100$ and its horizontal distance, measured in feet, is $x(t) = 8t$, where t is the time which has elapsed since the ball was thrown measured in seconds. What is the position of the ball when it has reached its highest point?

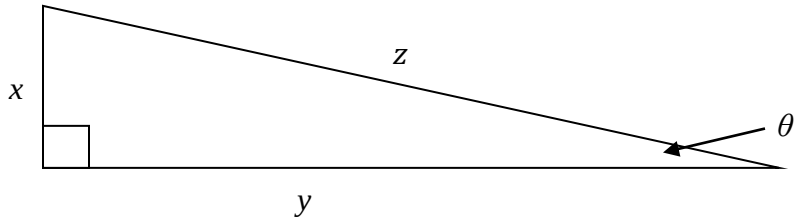
a) (1ft, 105ft)

b) (105ft, 8 ft)

c) (1 ft, 8 ft)

d) (8 ft, 105 ft)

Problems 34-36 refer to the right triangle below.



34) Which ratio is $\sin\theta$ for the given triangle?

a) $\frac{z}{x}$

b) $\frac{x}{z}$

c) $\frac{y}{z}$

d) $\frac{z}{y}$

35) The ratio $\frac{x}{y}$ represents which trigonometric value for θ?

a) $\csc\theta$
b) $\tan\theta$
c) $\cos\theta$
d) $\sin\theta$

36) Find the value of $\cos\theta$ if $x = 7$, $y = 24$, and $z = 25$. Round to two decimal places.

a) 0.29
b) 3.43
c) 0.96
d) 0.28

37) A person 150 ft from the base of a tree observes that the angle between the ground and the top of the tree is 30°. What is the height of the tree?

a) 75 ft
b) 86.60 ft
c) 129.90 ft
d) 259.81 ft

38) What is the measure of 140° in radians?

a) $\frac{2\pi}{3}$

b) $\frac{3\pi}{5}$

c) $\frac{6\pi}{7}$

d) $\frac{7\pi}{9}$

39) A teacher is celebrating a birthday. One of her students brings an apple as a present. The next student brings two apples. A third student brings four apples. The fourth student brings eight apples. If this pattern continues, how many apples will the 11^{th} student bring?

a) 1024
b) 2048
c) 512
d) 2047

40) A baseball team needs 9 players in order to play. There are 23 girls and 30 boys that have tried out for the teams. If there needs to be one girl's team, one boy's team, and each team must contain exactly 9 players, about how many different combinations of players can be chosen?

a) 14,307,150
b) 817,190
c) 116,916,599,085
d) 15,124,340

41) A target consists of a circle inscribed inside of a square. What is the probability of landing in the circle, assuming that any region of the square is as likely to be hit as any other region and assuming the objects must land within the square?

a) 0.637
b) 0.785
c) 0.215
d) 0.318

42) For an IQ test of individuals, the population mean is 100 and the population standard deviation is 15. What is the standard deviation of the sample mean with a sample size of 5? Assume that the population is normally distributed.

a) $\frac{15}{\sqrt{4}}$
b) 15
c) 3
d) $\frac{15}{\sqrt{5}}$

43) A map has a scale factor of 500 miles for every inch. The distance between Marley and Devon is 24 inches on the map. How far apart, in miles, is Marley from Devon?

a) 12
b) 120
c) 1200
d) 12000

44) A graph of experimental data showing a trend with a negative slope will have a correlation coefficient, r that is

a) $0 < r < 1$
b) $r < -1$
c) $-1 < r < 0$
d) $r > 1$

45) The confidence intervals for the population means of random variables A and B have the same width. A and B are distributed normally. The standard deviation of A is less than the standard deviation of B. Which statement must be true?

a) The sample size was larger for B, or the confidence level was less for B, or both
b) The sample size was larger for A, or the confidence level was less for A, or both
c) The sample size was larger for B, or the confidence level was larger for B, or both
d) The sample size was larger for A, or the confidence level was larger for B, or both

46) Which of the following survey methods would most accurately measure the food preferences of the student population in a school?

a) An electronic survey where students can complete a questionnaire on the web at their convenience
b) Individual interviews of friends and acquaintances of the surveyors
c) A paper survey available in the cafeteria during lunches
d) Individual interviews of a sample of students chosen at random from the school roster

47) Which of the following characteristics is shown on a modified boxplot?

a) A box with a width of two standard deviations
b) A bar in the box at the mean
c) Whiskers extending three standard deviations from the mean
d) Outliers, if they exist, beyond the whiskers

48) An organization has a calling tree to contact its members quickly and efficiently. In the first level, the first person calls four members. In the second level each of the four people called in level one call four other members. If this pattern continued how many people would be called after n levels?

a) $\sum_{i=1}^{n} i^4$
b) 4^n
c) $\sum_{i=1}^{n} 4^i$
d) n^4

49) Given that AD is a median of ▲ABC, what conclusion can be made?

a) AC=AB
b) BD=CB
c) CD=BD
d) AD=AB

50) ▲ABC and ▲DEF are similar. What must be true in order to show the triangles are congruent?

a) AB=DF
b) BC=DE
c) AB=AC
d) FE=CB

51) You want to prove that if a puppy is black then the puppy is male. What must be assumed if using a proof by contradiction?

a) The puppy is black and the puppy is not male.
b) The puppy is not black and the puppy is not male.
c) The puppy is black and the puppy is male.
d) The puppy is not black and the puppy is male.

52) Anaya has three meters of ribbon. She wants to make bracelets for her friends. If each bracelet must be 4 inches long, how many bracelets can she create? (There are 2.54 cm per inch.)

a) 22
b) 29
c) 30
d) 54

53) The altitude of a ▲ABC triangle is 4 inches. If the altitude of ▲DEF, a similar triangle, is double the altitude of ▲ABC, what is the ratio of the area of ▲DEF to the area of ▲ABC?

a) ¼ : 1
b) ½ : 1
c) 2 : 1
d) 4 : 1

54) Given the vertices of a triangle, what is the best way to determine it the triangle is right?

a) Use the distance formula
b) Use the Pythagorean Theorem
c) Use the distance formula and the Pythagorean Theorem
d) Cannot be done with the information provided

55) Jenna is converting 50 kilograms into ounces. One kilogram is equivalent to 2.2 pounds. She multiplies 50 by 2.2 and divides the product by 16. What is her mistake?

a) Multiplying by 2.2
b) Dividing by 16
c) Did not multiply by 1000
d) Jenna found the answer correctly

MORE ADVANCED PROBLEMS

56) How many permutations of the letters ABCDEFGH contain the string ADE?

a) 720
b) 40,320
c) 6,720
d) 120

57) A club has 12 female members and 14 male members. How many different ways can a committee of 6 be formed if an equal number of men and women must be selected?

a) 584
b) 80,080
c) 2,882,880
d) 3,504

58) Use set builder notation to give a description of the set $\{-1,0,1\}$

a) $\{x \in \mathbb{Z} | -1 \leq x \leq 1\}$
b) $\{x \in \mathbb{Z} | x^2 < 4\}$
c) $\{x \in \mathbb{Z} | -5 \leq x^3 \leq 6\}$
d) All of the above

59) Does Figure 1 have an Euler circuit?

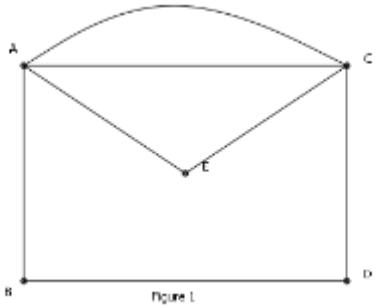


Figure 1

a) No

b) Yes, ABDCEACA

c) Yes, ECABD

d) Yes, ACDBAEC

60) Find the length (or weight) of the minimum spanning tree of the figure.

a) 4

b) 11

c) 8

d) 7

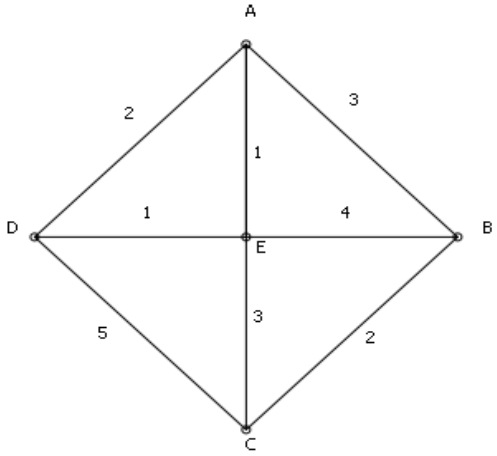


Figure 2

61) Consider the sets S and R below.

$S = \{x|\ x \text{ is an integer}\}$

$R = \left\{x|\ x \text{ is in the form of } \frac{a}{b} \text{ where } a \text{ and } b \text{ are integers and } b \neq 0\right\}$

Which of the following statements about S and R is true?

a) $S \cap R$ is the set of integers

b) $S \cup R$ is the set of complex numbers

c) $S \cap R$ is the set of complex numbers

d) $S \cup R$ is the set of irrational numbers

62) Which of the following could be disproven by the use of a counterexample?

a) $\forall x \in \mathbb{Z}\ (x^2 \geq x)$

b) $\forall x\ \in \mathbb{Z}\ (x > 0 \vee x \leq 0)$

c) $\forall x \in \mathbb{Z}(x^2 + 1 \geq 1)$

d) $\forall x \in \mathbb{Z}(x^3 \geq 0)$

63) Let $L(x, y)$ be the statement "x likes y". $\forall x \exists y$: $\sim L(x, y)$ can be translated to English as:

a) There is someone whom no one likes

b) Nobody likes everybody

c) Everybody likes someone

d) Someone likes everybody

64) For integers *a* and *b*, "$a \mid b$" means there exists an integer *q* such that $b = aq$. "$a \nmid b$" means that *b* can"t be expressed as *aq* for any integer *q*. Which of the following statements is true?

a) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $a \nmid c$ then $a \nmid bc$

b) Let $a, m \in \mathbb{Z}$. If $a|m$ then $a \equiv 0 (\mathrm{mod}\ m)$.

c) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $a|c$ then $a|(b + c)$.

d) Let $a, b, c \in \mathbb{Z}$. If $a|bc$ and $a \nmid c$ then $a|b$.

65) Evaluate the following $\int \frac{4x^3+2}{x^4+2x} dx$

a) $\frac{5(x^4+2x)}{x^5+5x^2} + C$

b) $e^{x^4+2x} + C$

c) $\ln|4x^3 + 2| + C$

d) $\ln|x^4 + 2x| + C$

66) In the figure, $\overline{AB}$ and $\overline{CD}$ are diameters of the circle with center O, $\overline{AB} \perp \overline{CD}$, and chord $\overline{DF}$ intersects $\overline{AB}$ at E. If $\overline{DE} = 6$ and $\overline{EF} = 2$, then the area of the circle is

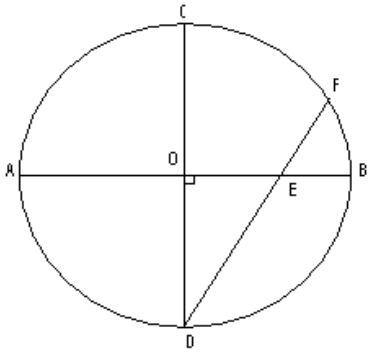
a) 23π

b) $\frac{47}{2}\pi$

c) 24π

d) $\frac{49}{2}\pi$

e)



67) A conic section is tangent to the x-axis at $x = 4$ and the y-axis at $y = -2$, which of the following equations may be used to model the conic section?

a) $x^2 + 4y^2 - 8x + 16y + 16 = 0$

b) $x^2 + 2y^2 - 8x + 8y + 20 = 0$

c) $x^2 + 4y^2 + 8x - 16y + 16 = 0$

d) $4x^2 + y^2 + 16x - 8y + 20 = 0$

68) Which of the following is <u>not</u> a 6th root of unity?

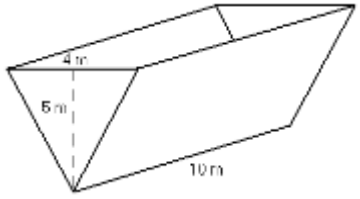
a) -1

b) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$

c) $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

d) $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$

69) Consider the isosceles trough shown to the right.



Suppose the trough is filled at a rate of 5 m^3/sec. At what rate is the height changing when the water is at a height of 3 meters?

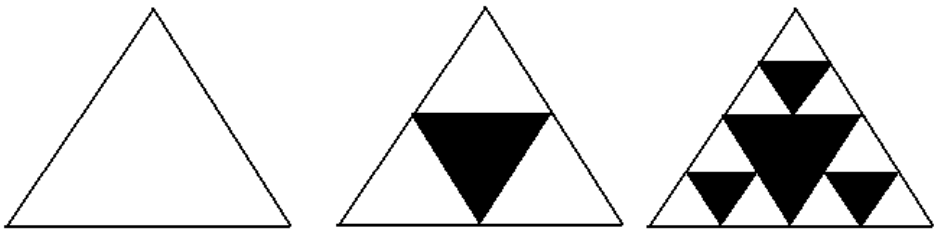
a) $\frac{5}{12}\ m/_{sec}$

b) $\frac{5}{48}\ m/_{sec}$

c) $\frac{5}{6}\ m/_{sec}$

d) $\frac{5}{24}\ m/_{sec}$

70) Consider the pattern below, in which the first triangle is equilateral with area 1, and all shaded regions are equilateral triangles.



If the pattern is continued, what is the area of the shaded region for the nth iteration?

a) $\sum_{k=1}^{n} \frac{1}{4}\left(\frac{3}{4}\right)^{k-1}$

b) $\frac{1}{4} + \left(\frac{3}{4}\right)^{n}$

c) $\sum_{k=1}^{n} \left(\frac{1}{4}\right)^{k}$

d) $\left(\frac{1}{4}\right)^{n}$

71) Suppose a vector $A = 2\hat{e}_\rho + 3\hat{e}_\phi + \hat{e}_z$ is given in cylindrical coordinates at the point $(\rho, \phi, z) = \left(1, \frac{\pi}{2}, 2\right)$. Which of the following is a vector representation of A in Cartesian coordinates?

a) $-3\hat{e}_x + 2\hat{e}_y + \hat{e}_z$

b) $2\hat{e}_x + 3\hat{e}_y + \hat{e}_z$

c) $\hat{e}_x + 3\hat{e}_y + \hat{e}_z$

d) $-3\hat{e}_x + 2\hat{e}_z$

72) Which of the following sets could <u>not</u> represent a basis for a linear vector space?

a) $\{(1,0,0)(1,1,0)(1,1,1)\}$

b) $\{1,\ x,\ x^2,\ x^3,\ x^4\}$

c) $\{\sin mx : m \in \mathbb{Z}^+\}$

d) $\{a, 2a, 3a, 4a\}$